المجموعات فوق شبه المفتوحة من النوع α

On supra semi-α-open sets

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ملخص

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الملخص

في هذا البحث قدمنا مفهوما لنوع جديد من المجموعات المفتوحة في الفضاء التبولوجي تدعى المجموعة فوق شبه المفتوحة من النوع α .

ومن خلال هذا المفهوم سوف ندرس بعض أنواع من الدوال المستمرة وموضوعات الفصل.

الكلمات المفتاحية : المجموعات المفتوحة من النوع α, المجموعات شبه المفتوحة من النوع α,المجموعات فوق المفتوحة من النوع α, الفضاءات فوق التبولوجية, الدالة المستمرة من النوع α, الفضاء semi-α-T_0, الفضاء semi-α-T_1 و الفضاء semi-α-T_2.

Abstract

On supra semi-α-open sets

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In this paper we introduce a concept fora new type ofopen setsintopologicalspacecalledthe supra semi-α-open set.
Through this conceptwewillexaminesome typesofcontinuousfunctionsandseparation axioms.

Key words : α-open sets, semi-α-open, supra-α-open sets, supra topological spaces ,α-continuous functions, semi-α-T_0 space, semi-α- T_1 space and semi-α-T_2 space.

3.3.3 Example :

If we look at theexample **3.3.2**we find that theX supra semi-α-T_2 spacebut it is not T_2 space , because 1 ≠2There are no two setsareopen, such as T,G so, onecontainingtheelement1and the othercontainingthe element2and $T \cap G = \varnothing$.

References :

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77-84.

3.3.1 Remark:

Every T_0 space is supra semi-α-T_0 space.

The opposite situation is not necessarily true.

3.3.1 Example :

Let X={1,2,3}, $\tau_X = \{\emptyset, X, \{3\}\}$ Then $\tau_X^* = \{\emptyset, X, \{3\}, \{1,3\}, \{2,3\}\}$

We note that X is supra semi-α-T_0 space but not T_0 because 1≠2There are no opensetso thatcontainonewithouttheother.

3.3.2 Remark :

Every T_1 space is supra semi-α-T_1 space .

The opposite situation is not necessarily true .

3.3.2 Example :

Let X={1,2,3}, $\tau_X = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$ Then
$\tau_X^* = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$

We note that X is supra semi-α-T_1 space but not T_1 because

2≠3There are no two setsareopenso thatoneofthemcontainingelement 2andtheothercontainingtheelement 3.

3.3.3 Remark :

Every T_2 space is supra semi-α-T_2 space .

The opposite situation is not necessarily true .

Let X be supra semi-α-T_0 space, x $\neq$y in X

Hence there exists G an supra semi-α- open set in X such that x $\in$ G, y $\notin$ G or x $\notin$ G, y $\in$ G

Then G^c is supra semi-α-closed set and x $\notin G^c$, y $\in G^c$

Therefore x $\notin$ss cl_{α} {y} (since x $\notin G^c$)

Hence ss cl_{α} {x} $\neq$ss cl_{α} {y} .

3.3.1 Proposition:

A space X is supra semi-α-T_1 space if and only if {x} is supra semi-α-closed set, $\forall$ x $\in$X.

Proof:

$\Rightarrow$Let X be supra semi-α-T_1 space.

Let p $\in$ X, to prove {p} is supra semi-α-closed set.

X $\in$ $\{p\}^c$= X\{p} $\Rightarrow$ x $\neq$ p in X

Hence there exists an supra semi-α-open set G such that x $\in$G, p $\notin$ G or x $\notin$ G, p $\in$ G.

If x$\in$G, p$\notin$G $\Rightarrow$ x $\in$ G $\subseteq$ $\{p\}^c$$\Rightarrow$ $\{p\}^c$ is an supra semi-α-open set $\Rightarrow$ {p} is supra semi-α-closed set.

$\Leftarrow$Let {p} be an supra semi-α-closed set, $\forall$ p $\in$ X, to prove X is supra semi-α-T_1 space.

Let x $\neq$y in X

Hence {x}, {y} are supra semi-α-closed sets $\Rightarrow$ $\{x\}^c$, $\{y\}^c$are

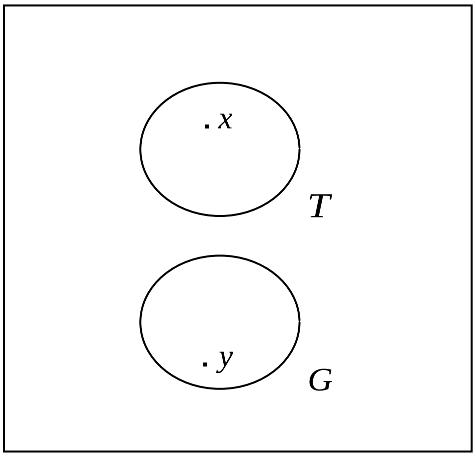
Supra semi-α- open sets and y $\in$ $\{x\}^c$, x $\notin$ $\{x\}^c$, x $\in$ $\{y\}^c$, y $\notin$ $\{y\}^c$

Therefore X is supra semi-α-T_1 space.

Supra semi-α-T_1 space

3.3.3 Definition :

A space X is said to be supra semi-α-T_2 space if for each pair of distinct points x , y in X there exist two supra semi-α - open sets T and G such that $x \in$ T , $y \in$ G and T $\cap$ G = $\emptyset$.



Supra semi-α-T_2 space

3.3.1 Theorem:

A space X is a supra semi-α-T_0 space if and only if sscl_α {x} ≠sscl_α {y} for each x ≠ y in X.

Proof:

Let sscl_α {x} ≠sscl_α {y} for each x ≠ y in X

Hence sscl_α {x} $\not\subseteq$ sscl_α {y} or sscl_α {y} $\not\subseteq$ sscl_α {x}

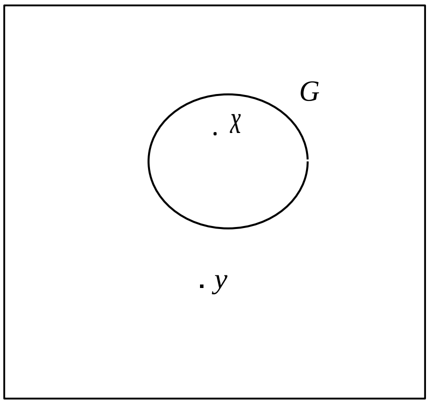
Suppose that sscl_α {x} $\not\subseteq$ sscl_α {y} ⇒x ∉sscl_α {y} ⇒ x ∈ $(sscl_\alpha\{y\})^c$

but $(sscl_\alpha\{y\})^c$ is an supra semi-α-open set and y ∉ $(sscl_\alpha\{y\})^c$

Therefore X is α-T_0 space.

3.3.1 Definition:

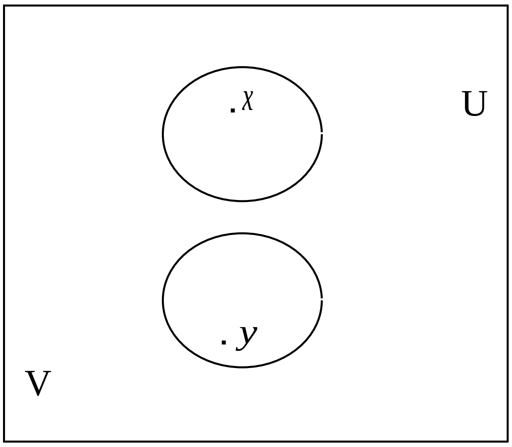
A space X is said to be supra semi-α-T_0 space if for each pair of distinct points x , y in X there exist G supra semi-α -open set of X containing one point but not the other.



Supra semi-α-T_0 space

3.3.2 Definition :

A space X is said to be supra semi-α- T_1 space if for each pair of distinct points x , y in X there exist two supra semi-α-open sets U and V containing x and y respectively, such that $y \notin$ U , $x \notin$ V



3.2.2 Theorem:

Every supraα-continuous function is a supra semi-α-continuous function .

Proof:

Let $f : X \to Y$ be a supraα-continuous function .Therefore $f^{-1}(A)$ is a supra α-open set in X for each open set A in Y.

Since : Every supra α-open set is supra semi-α-open set , This implies $f^{-1}(A)$ is supra semi-α-open set in X.

Hence f is a supra semi-α-continuous function.

The converse of the above theorem need not be true. This is show by the following example.

3.2.2 Example :

Let X={a,b,c}, $\tau_X = \{\varnothing, X, \{a\}, \{b\}, \{a,b\}\}$

$\tau_X^* = \tau_X$ Then

Let $f : X \to X$ where : $f(a) = a, f(b) = f(c) = b$

supra semi-α-continuous function but not supra α-continuous f

because {b} is open set in X and $f^{-1}(\{b\} = \{b,c\})$

But $\{b,c\}$ is not supra α-open set in X .

3.3 Supra semi-α-T_i spaces , $i = 0,1,2$

In this section we introduce a new class of separation axioms.

3.2.2 Definition:

Let (X,τ) and (Y,σ) be two topological spaces and τ^*,σ^* are associated supra topology with τ and σ respectively.
we define a function $f : X \to Y$ to be a supra semi-α-continuous function if the inverse image of each open set in Y is supra semi- α-open set in (X,τ^*).

[5] 3.2.1Theorem:

Every continuous function is a supra α-continuous function

The converse of the above theorem need not be true. This is show by the following example.

3.2.1 Example:

Let X={a,b,c,d}, $\tau_X = \{\emptyset, X, \{a\}\}$

$$\tau_X^* = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

Let Y={x,y,z}, $\tau_Y = \{\emptyset, Y, \{x\}\}$

$$\tau_Y^* = \{\emptyset, Y, \{x\}, \{x,y\}, \{x,z\}\}$$

Let $f : X \to Y$ where: $f(a) = f(b) = x,\ f(c) = y,\ f(d) = z$

supraα-continuous function but not continuous function because f

{x} is open set in Y and $f^{-1}(\{x\} = \{a,b\}$

But $\{a,b\}$ is not open set in X .

The supra semi-α-interior of a set A is denoted byss $\operatorname{int}_{\alpha}(A)$ and defined as :ss
$\operatorname{int}_{\alpha}(A)$ = $\bigcup$ {B:B is a supra semi- α-open set and $A \supseteq B$ }.

3.1.1 Theorem:

Any union of supra semi-α-open set is always a supra semi- α-open set.

Proof:

Let { A_i } $_{i\in J}$ family of supra semi-- α-open set , this is

Supra α-open set such that : $G_i \subseteq A_i \subseteq$ supra cl(A_i) ; $\forall i \in J$ $\exists$ G_i $\in$

supra cl ($\Rightarrow$ $\bigcup G_i \subseteq \bigcup A_i \subseteq \bigcup A_i$)

supra cl ($\bigcup A_i$)$\subseteq$

supra cl ($\bigcup A_i$)$\Rightarrow$ $\bigcup G_i$ $\subseteq$

supra semi- α-open set.$\Rightarrow$ $\bigcup${ A_i } $_{i\in J}$

3.1.1 Corollary :

Any intersection of supra semi- α-closed set is always a supra semi- α-closed set.

3.2 Supra semi-α-continuous function

In this section we introduce a new class of functions .

3.2.1[5] Definition:

Let (X,τ) and (Y,σ) be two topological spaces and τ^*,σ^* are associated supra topology with τ and σ respectively.
we define a function $f : X \to Y$ to be a supra α-continuous function if the inverse image of each open set in Y is supra α-open set in (X,τ^*) .

Complement of a supra semi- α-open set is called a supra semi-α-closed set.

3.1.1 Remark:

Every supra open set is supra α-open set

The following example shows that the converse of the above remark is not true.

3.1.1 Example :

Let (X,τ^{*}) be a supra topological space where X={a,b,c} and τ^{*}= {∅ , X ,{a}}. Here ,{a,b} is a supra α-open set but not supra open set.

3.1.2 Remark:

Every supra α-open set is supra semi-α-open set

The following example shows that the converse of the above remark is not true.

3.1.2 Example :

Let (X,τ^{*}) be a supra topological space .where X={a,b,c,d} and τ^{*} = {∅, X ,{a},{b},{a,b},{a,b,c}}.
Here , {b,c} is a supra semi-α-open set , but not a supra α-open set .
From the remarks above we get the following :

Supra open set ⇒ supraα-open set⇒ supra semi-α-open set

3.1.2 Definition :

The supra semi-α- closure of a set A is denoted by ss$cl_{\alpha}(A)$ and defined as :ss $cl_{\alpha}(A)$ = ⋂ {B:B is a supra semi- α-closed set and $A \subseteq B$ }

[2] 2.9 Definition:

Let (X,τ) be a topological space and τ^* be a supra topology on X . we call τ^* a supra topology associated with τ if $\tau \subset \tau^*$.

[2] 2.10 Definition:

Let (X,τ) and (Y,σ) two be a topological spaces, Let τ^* and σ^* are associated supra topologies with τ and σ respectively.

Let $f : X \to Y$ be a map from X into Y , then f is a supra continuous function if the inverse image of each open set in is supra open set in (X,τ^*).

2.11 Definition:

Let $f : X \to Y$ be a function, then f is said to be Semi-α-continuous function if and only if for each A open set in Y, then $f^{-1}(A)$ is a semi-α-open set in X.

3- Results and discussions :

3.1 Basicproperties of supra semi-α-open sets

In this section we introduce a new class of sets.

3.1.1 Definition:

Let (X,τ^*) be a supra topological space . A set A is called :

(i) supra α-open set if :
A $\subseteq$ supra int(supra cl (supra int(A))).
Complement of a supra α-open set is called a supra α-closed set.

(ii) supra semi-α-open set if:

U $\subseteq$ A $\subseteq$ supra cl(A) , and U is supra α-open set in (X,τ^*)

[4] 2.4 Definition:

Let (X,τ) be topological space .A set A is called semi-α-open set if U $\subseteq$ A $\subseteq$ cl(U) and U isα-open set in (X,τ).
Complement of a semi-α-open set is called a semi-α-closed set.

[4] 2.5 Definition :

A space X is said to be semi-α-T_0 space if for each pair of distinct points in X there exist semi-α-open set of X containing one point but not the other.

[4] 2.6 Definition:

A space X is said to besemi-α-T_1 space if for each pair of distinct points x , y in X there exist a two semi-α-open sets U and V containing x and y respectively, such that $y \notin$ U , $x \notin$ V .

[4] 2.7 Definition :

A space X is said to be semi-α-T_2 space if for each pair of distinct points x , y in X there exist two a semi-α-open sets G_1 and G_2 such that

$x \in G_1$, $y \in G_2$ and $G_1 \cap G_2 = \varnothing$.

[2] 2.8 Definition :

The supra closure of a set A is denoted by scl(A) and defined as:

scl(A) = $\bigcap$ {B:B is a supra closed set and $A \subseteq B$ }.

The supra interior of a set A is denoted by sint(A) and defined as:

sint(A) = $\bigcup$ {B:B is a supra open set and $B \subseteq A$ }.

1-Introduction:

In 1965, Najastad [1] introduced the α-open sets.
In 1983, A.S.Mashhour [2] introduced the supra topological spaces and studied supra continuous function.
In 1985, I.Reilly and M.vamanamurthy[3]introduced α-continuous functions.
In 2000,G.B.Navalagi[4] introduced the semi-α-open sets.
In 2008,R. Devi ,S.Sampathkumar and M.Calads[5] introduced the supra α-open sets and Sα-continuous function.

Now, we introduce the concept of supra semi-α-open sets and supra semi-α-continuous and investigate some of the basic properties for this class of functions and study some types of separation axioms in topological spaces.

2. Definitions and concepts :

2.1[1] Definition:
Let (X,τ) be topological space .A set A is called α-open set if :
A $\subseteq$ int (cl(int(A))).
Complement of a α-open set is called a α-closed set.
[2] 2.2 Definition:
A subfamily τ^* of X is said to be a supra topology on X if:

(1) $X, \emptyset \in \tau^*$.

(2) if $A_i \in \tau^*$ for all $i \in J$, then $\bigcup A_i \in \tau^*$.

is called a supra topological space . (X,τ^*)

The elements of τ^* are called supra open sets in (X,τ^*) and complement of supra open sets is called a supra closed set.

[3] 2.3 Definition:

Let (X,τ) and (Y,σ) be two topological spaces .A function $f : X \to Y$ is called α-continuous function if the invers image of each open set in Y is α-open set in X.